ENERGY PROPAGATION IN DIELECTRIC AND MAGNETIC MATERIALS : THEORETICAL NOTES, DEFINITIONS AND CALCULATIONS

The following theoretical notes are based on sources such as Stratton "Electromagnetic Theory", McGraw Hill Book Company and Von Hippel "Dielectric Materials and Applications", Technology Press of M. I. T. and John Wiley and Sons (reprinted by Artech House).

The formulas and discussions are presented here for the convenience of the designer planning to use Dielectric Materials. Only the most practical formulas here have been selected, and an attempt has been made to simplify those presented. On the other hand, the use of many of the formulas requires the ability to manipulate complex numbers.

It is important to note that the formulas presented here apply, with proper interpretations, equally well to:

1. a flat sheet in free space,
2. a slug which completely fills a section of coaxial transmission line, and
3. a rectangular solid which completely fills a rectangular wave guide operating in the TE_{10} mode.

DEFINITIONS

1. ***Complex Dielectric Constant***

$$K^* = K - jK\tan\delta_d = \frac{\varepsilon'}{\varepsilon_0} - j\frac{\varepsilon'}{\varepsilon_0}\tan\delta_d$$

where K is the real part of the relative permittivity $\frac{\varepsilon'}{\varepsilon_0}$, and $\tan\delta_d$ is the dielectric loss tangent.

2. ***Complex Relative Magnetic Permeability***

$$K^*_m = K_m - jK_m\tan\delta_m = \frac{\mu'}{\mu_0} - j\frac{\mu'}{\mu_0}\tan\delta_m$$

where K_m is the real part of the relative permeability $\frac{\mu'}{\mu_0}$, and $\tan\delta_m$ is the magnetic loss tangent.

d = thickness of flat sheet in free space or length of slug along the transmission line, either coaxial line or waveguide.

λ_0 is free space wavelength.

θ is incidence angle, the angle between the direction of propagation of signal impinging on the free space slab and the normal to the free space slab; θ is always zero for the coaxial line; for wave guide in the TE_{10} mode.

$$\theta = \cos^{-1}\frac{\lambda_0}{\lambda_g}$$

where λ_g is the wave guide wavelength and is given by

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}}$$ where air or vacuum is filling the waveguide but

$$\lambda_g = \mathrm{Real}\left[\frac{\lambda_0}{\sqrt{K^*K^*_m - \left(\frac{\lambda_0}{2a}\right)^2}}\right]$$ where material is filling the waveguide

where a is the broad dimension of the waveguide.

3. ***Impedance, Relative Impedance,***

$$\text{Impedance of free space} = Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$$

Intrinsic impedance of a material = $Z = \sqrt{\frac{\mu^*}{\varepsilon^*}} = \sqrt{\frac{\mu' - j\mu'\tan\delta_m}{\varepsilon' - j\varepsilon'\tan\delta_d}}$

Relative impedance magnitude or absolute value of relative impedance =

$$\frac{|Z|}{Z_0} = \frac{|Z|}{377} = \left|\sqrt{\frac{\frac{\mu'}{\mu_0} - j\frac{\mu'}{\mu_0}\tan\delta_m}{\frac{\varepsilon'}{\varepsilon_0} - j\frac{\varepsilon'}{\varepsilon_0}\tan\delta_d}}\right| = \left|\sqrt{\frac{K_m - jK_m\tan\delta_m}{K - jK\tan\delta_d}}\right|$$

4. ***Attenuation in dB/cm.***

The following equation may be used to calculate the attenuation constant for any material in terms of its dielectric and magnetic properties:

Attenuation constant = $\frac{dB}{cm} = \frac{2\pi(8.686)}{\lambda_0}\sqrt{\frac{K'K'_m}{2}\left[\sqrt{(1+\tan^2\delta_d)(1+\tan^2\delta_m)} - (1-\tan\delta_d\tan\delta_m)\right]}$

where λ_0 = free-space wavelength in cm

The attenuation values tabulated or computed from the equation must be interpreted as in the following example. Suppose that a slug of lossy material completely fills a section of a coaxial transmission line. Once energy has entered the lossy material, it will be diminished at the given rate as it propagates in the section of line that is filled by the material. The tabulated or calculated rate does not include any effects of energy reflections at the input or output interfaces between the lossy material and any other materials such as air in adjacent sections of the line.

5. ***Polarization***

Before presenting transmission and reflection formulas, the two principal polarization cases for electromagnetic impingement on a flat sheet must be defined. These are perpendicular and parallel polarization, identified respectively in the formulas by the subscripts $\perp$ and $||$.

The terms refer to the direction of the electric field in a linearly polarized wave. They also refer to an incidence plane which is the plane formed by two lines, one of which is the normal to a flat sheet of material on which the wave is impinging, and the other which is the direction of propagation of the impinging wave. The incidence angle, θ previously defined lies in this incidence plane. In a coaxial line the incidence angle is zero; therefore polarization does not apply. In the TE_{10} waveguide mode, the polarization is always perpendicular.

6. ***Interface Voltage Reflection Coefficients***

There are two interface voltage reflection coefficients (r) one for each polarization:

a. for perpendicular polarization

$$r^*_\perp = \frac{K^*_m \cos\theta - \sqrt{K^*_m K^* - \sin^2\theta}}{K^*_m \cos\theta + \sqrt{K^*_m K^* - \sin^2\theta}}$$

b. for parallel polarization

$$r^*_\parallel = \frac{\sqrt{K^*_m K^* - \sin^2\theta} - K^* \cos\theta}{\sqrt{K^*_m K^* - \sin^2\theta} + K^* \cos\theta}$$

Electrical Thickness, ϕ

This is the same for both polarizations and is given by

$$\phi^* = \frac{2\pi d}{\lambda_0}\sqrt{K^*_m K^* - \sin^2\theta}$$

At normal incidence in free space and in a coaxial line, the last three formulas are simplified by the incidence angle, θ being zero and become:

$$r^*_\perp = \frac{\sqrt{K^*_m} - \sqrt{K^*}}{\sqrt{K^*_m} + \sqrt{K^*}} = r^*_\parallel$$

and

$$\phi^* = \frac{2\pi d}{\lambda_0}\sqrt{K^*_m K^*}$$

7. Transmission and Reflection

In terms of the interface reflection coefficient r, and electrical thickness ϕ . We can now write the transmission and reflection equations for the flat sheet in free space, the solid slug in a coaxial line, and the rectangular solid in the waveguide.

8. Voltage Transmission Coefficient, T*

$$T^* = \frac{\left(1 - r^{*2}\right)e^{-j\phi^*}}{1 - r^{*2}\, e^{-j2\phi^*}}$$

from which the insertion loss in decibels is given by

$$10\log\frac{1}{|T^*|^2}$$

and the insertion phase delay introduced by the sample over the distance occupied by the sample is given by

$$T' - \frac{2\pi d}{\lambda_0}\cos\theta$$

where T’, is the argument of T*.

9. Voltage Reflection Coefficient, R*

$$R^* = \frac{-r^*\left(1 - e^{-j2\phi^*}\right)}{1 - r^{*2}\, e^{-j2\phi^*}}$$

from which the reflection loss in decibels is given by

$$10\log\frac{1}{|R^*|^2}$$

and the reflection phase or delay due to energy lingering in the sample prior to reflection is simply R', the argument of R*.

10. *Metal Backed Reflection Coefficient*

If the dielectric material is backed by metal or short circuited, the reflection coefficient becomes:

$$R^*_{sc} = \frac{r^* - e^{-j2\phi^*}}{1 - r^* e^{-j2\phi^*}}$$

in which again reflection loss is given by

$$10\log\frac{1}{|R^*_{sc}|^2}$$

11. *Open Circuit Reflection Coefficient*

If a quarter wave of air is placed between the dielectric material and the short circuit, the material is said to be open circuited, and

$$R^*_{oc} = \frac{r^* + e^{-j2\phi^*}}{1 + r^* e^{-j2\phi^*}}$$

and again reflection loss is given by

$$10\log\frac{1}{|R^*_{oc}|^2}$$

This device is often used to take full advantage of the reflection loss potential in a lossy dielectric used for a termination.

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